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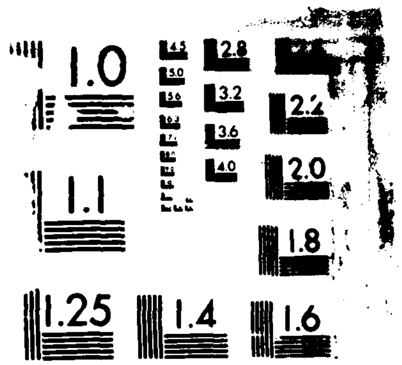

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Thesis

Robert R. Fisher
Capt USAF

AFIT/GA/AA/86D-5

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Astronautical Engineering

Robert R. Fisher
Captain, USAF

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PREFACE

I extend my sincerest gratitude to Dr. William Wiesel, my thesis advisor, for his help and support during the past year. Dr. Wiesel's knowledge of astrodynamics and optimization theory, and his willingness to share this knowledge made the completion of this thesis possible. I hope that my two daughters, Elizabeth and Katherine, will forgive me for the lack of attention they have had to endure these last eighteen months. I wish to express my thanks especially to my most loving wife and partner, Marie, who gave me the psychological and moral support I needed to complete this project.

Robert R. Fisher

Computer: Zenith Z–150

Software: Microsoft Word

Softcraft Fancy Word

Printer: Epson RX–80

TABLE OF CONTENTS

List of Figures

List of Symbols

Symbol

A – average area of the External Tank

a – semi–major axis of an orbit

C_i – optimization coefficient for tether tension profile

D – distance between the shuttle and the ET

D_0 – initial separation between the shuttle and the ET

$\mathcal{E}_s$ – orbital energy of the shuttle

$\mathcal{E}_{s0}$ – orbital energy of the shuttle for the nominal tether force

$\underline{F}_D$ – force of air drag on the external tank

F_D – magnitude of air drag on the external tank

$\underline{F}_T$ – tension force on the tether

F_T – magnitude of the tension force on the tether

h – height above the surface of the Earth

h_{ref} – reference value for use in atmospheric model

$\mathcal{L}$ – Lagrangian for the system dynamics

X, Y, Z – unit vectors in an inertial geocentric frame

m_s – Mass of the Space Shuttle

m_t – mass of the external tank (ET)

Q_i – Lagrangian notation for the generalized forces acting on the system

q_i – Lagrangian notation for the generalized coordinates

R_E – radius of the Earth

$\underline{R}_{ref}$ – position vector for the reference orbit

R_{ref} – magnitude of the position vector for the reference orbit

$\underline{R}_{srel}$ – position vector for the shuttle expressed in the relative frame

R_{srel} – magnitude of position vector for the shuttle

$\underline{R}_{trel}$ – position vector for the tank expressed in the relative frame

R_{trel} – magnitude of position vector for the tank

$\mathcal{T}$ – kinetic energy of the two body system

$\mathcal{V}_g$ – gravitational potential energy

$\underline{V}_{srel}$ – inertial velocity of the shuttle expressed in the relative frame

V_{srel} – magnitude of inertial velocity of the shuttle

$\underline{V}_{trel}$ – inertial velocity of the ET expressed in the relative frame

V_{trel} – magnitude of inertial velocity of the ET

$\mathcal{V}_{TOT}$ – Total potential energy of two body system

$\underline{X}$ – column vector of tether tension coefficients

$\delta\underline{X}$ – column vector of corrections for tether tension coefficients

δr – radial coordinate direction for relative coordinate frame

$\delta\theta$ – downtrack angular coordinate direction for relative coordinate frame

δr_t – relative radial measurement for the external tank

δr_s – relative radial measurement for the space shuttle

$\delta\theta_s$ – relative downtrack angular measure for the space shuttle

$\delta\theta_t$ – relative downtrack angular measure for the external tank

ρ_a – atmospheric density at the ET altitude

ρ_t – mass density of tether connecting the ET and shuttle

ρ_0 – reference value for the atmospheric density in the exponential model

μ_e – gravitational parameter for the Earth

Ω – angular velocity of reference orbit of relative coordinate frame

Abstract

The possible use of tethers in space has been proposed for the last hundred years. While much work has been done recently on the use of tethers for towed satellites from the Space Shuttle, little has been done to determine the possible benefits of using tethers as propulsive devices to supplement or replace rocket engines for boost from Low Earth Orbit. This project attempts to determine one method of using tethers to improve the performance of the Space Shuttle. Orbit insertion parameters such as velocity and final altitude for the space shuttle are limited by operational constraints on the possible delta V that can be supplied from the engines. The possibility of increasing the performance of the shuttle exits by use of an inter–connecting tether to serve as a momentum transfer device between the External Tank and the Shuttle. This added momentum would widen the possible orbit options presently available by boosting the shuttle to a higher orbit. This project derives the equations of motion for a three–body connected dynamical system to include the Shuttle, the external tank, and the cable in orbit around a spherical Earth. Due to current material limitations the tether length will be limited to 100 kilometers. The possible envelope of orbital changes is investigated, and this program determines through an optimization routine the tension profile in the cable, and the initial separation distance to apply that tension that results in the maximum altitude gain for the shuttle.

OPTIMAL SHUTTLE ALTITUDE CHANGES USING TETHERS

I. Introduction

In the year 1895, the great Russian scientist Tsiolkovsky (23:165) first proposed connecting large masses in space by long cables to exploit the weak gravity gradient forces that exist in orbit, and thus establish "towers" in space. These large towers would reach from near the ground to out beyond geosynchronous altitude, and would be supported by the tension generated by excess centrifugal force on the higher portions of the structure that would extend beyond geosynchronous. An Earth–based version of this idea that would reach to low–earth orbit would require a super–strength material such as diamond (or other perfect crystals). However, the idea of using tethers to lift or propel objects in space does not require these super–materials. Space tethers made of such substances as Kevlar or stainless steel, could augment or even replace rockets for the transfer of payloads from low earth orbits to orbits farther out. The basic mechanism for this process is the simple exchange of momentum between the two bodies attached at the ends of the tether, one of them a carrier and the other the desired payload. If two objects are connected by a long tether and placed into an orbit that

is roughly circular, the minimum energy configuration for this dumbbell shaped system is with the tether aligned along the local vertical. In this arrangement, the tether supports the lower object (which moves with sub–orbital velocity) and retards the higher object (which moves with supra–orbital velocity). Momentum can be transferred between the two end bodies by lengthening or shortening the tether length from some nominal initial value. This change in tether length generates Coriolis forces that temporarily tilt the system away from the local vertical (see **Fig. 1**). The end result is that one mass of the system is decelerated while the other one is accelerated. The factor that determines which body is accelerated and which is decelerated is whether the tether is lengthened or shortened from

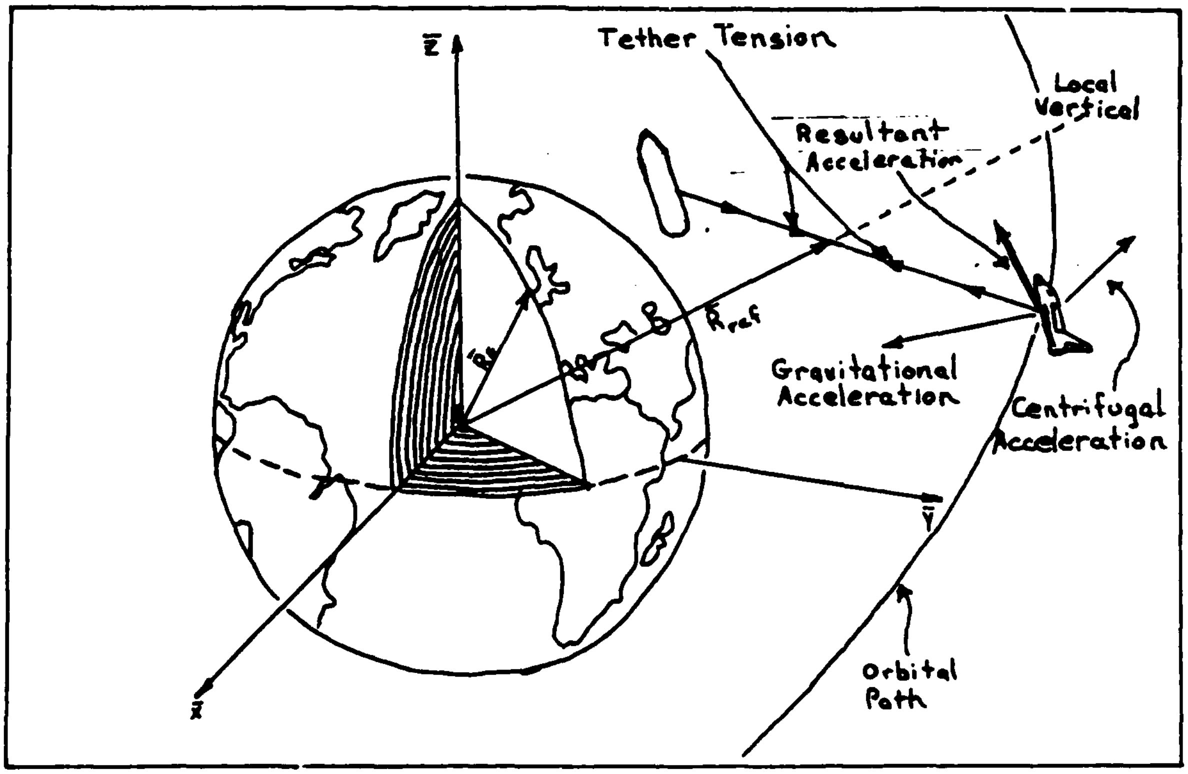

Figure 1 Displacement from Local Vertical

the original value. If the tether length increases after initial deployment of the tether, then the higher object will be accelerated when tension is applied to the cable and the lower body will be slowed down. Thus, the momentum of the lower body is transferred to the higher one to maintain the tether alignment along the local vertical.

An example of this momentum "toss" would be an exchange through a connecting tether between the shuttle and the external tank with the tank serving as the carrier and the shuttle serving as the payload. At the normal separation point, the shuttle is detached from the tank and acquires about an additional 20 fps of velocity. The increasing radius of the shuttle from this velocity increase causes it to appear to fall behind the external tank as it's elliptical path carries it into a higher orbit. The difference in angular velocities between the lower, faster moving tank, and the higher, slower moving shuttle cause this apparent relative velocity to occur. As the shuttle moves away and behind the jettisoned external tank, the connecting tether cable will pay out. Any displacement of the tether cable away from the local vertical when tension is applied to it will tend to create a force at the ends of the tether that try to realign it with the local vertical. When the system is aligned along the local vertical, the shuttle and tank have the same angular velocity, but the shuttle has a higher linear velocity due to it's larger orbital radius. When the tether length is increased from it's original value, the two end bodies will tend to move farther apart and the path of the orbital center of mass does not change. Yet, due to the coriolis forces that are generated the tether will still tend to align itself along the local vertical. As a direct result of this, the shuttle would gain in altitude, and the tank would lose altitude when the tether cable length was changed from

it's original value. When the cable is discarded or cut, the end bodies will drift apart into different elliptical orbits. Since the linear velocity of the shuttle at the moment of release is higher than that required for an orbit at the same altitude, the orbit that it enters will be elliptical with the perigee at the release point, and the apogee at the opposite side of the earth.

In order to have a controllable tether system that uses these benefits of tether behavior, it is usually necessary to have a tether deployer, controller, and retriever. A tether system has been developed for use on the Space Shuttle, not for a tether toss as described above, but to deploy and retrieve small scientific satellites (3,13,20,23) for use on low Earth orbit. The idea of flying scientific payloads suspended below the Space Shuttle has been developed by G. Columbo (12) of Italy for tethers up to 100 km long. The primary purpose of this satellite and cable system is to allow for the towing of satellites in the Earth's upper atmosphere for research into upper atmospheric physics. An extremely simple disposable tether "toss" system is under development at Energy Sciences Lab in San Diego, California, that would fit into a Getaway Special container in the Shuttle's cargo bay (23:168). The system has almost no moving parts with the tether unwinding from a stationary spool. The tension between the Shuttle and the deploying tether is maintained by inflatable gas bags that squeeze the line between flaps during deployment. A meter measures the amount of line deployed and a cutter severs the tether when the right amount of line has been released. According to studies performed, this would allow the deployment of a 350 pound payload using a ten kilometer long tether. In a simple case as described above the retriever portion of the package may be deleted.

Since the tension in the tether cable is proportional to the length of the tether, the limiting factor in the use of tethers becomes the amount of tension the material strength of the cable can withstand. The limiting load of the tether is determined by the properties of the material, as well as the diameter of the cable material being used. Thus if the tether needs to be stronger to withstand the loads imposed during the + ther "toss", one method of solution is that of brute force. Simply increase the diameter of the cable until it can withstand the imposed loads. Increasing the diameter of the tether does not always solve this material limitation because the tether must still support itself, and since the weight of the tether increases linearly with the length of the tether it will eventually reach a point where the cable could not support itself for the lengths required. If the cable tether was made of Kevlar, the maximum length due to material properties of a tether is approximately 480 kilometers in Low Earth Orbit (LEO) (23:168). When the tether exceeds this length, the mass of the tether alone causes it to exceed the material strength of the cable using only a nominal safety factor. It is possible to partially work around the physical limitations of the tether by using a tether that is tapered at the ends. When the tether is tapered the portion of the tether near the endpoints only needs to be strong enough to support the forces on the end bodies. However, the portion of cable at the dumbbell–type system's center of mass must still be strong enough to support the forces generated at the endpoints as well as the mass of the tether.

For a simple tether deployment system between the shuttle and the external tank, the tether retrieval portion may by discarded from the tether control system. This project will attempt to determine the optimum tension

profile to maintain in the tether cable as it is deployed between the external tank and the shuttle so as to maximize the orbital altitude gain of the shuttle from the momentum "toss". Several simplifying assumptions will be made to limit the scope of this problem. These assumptions are:

1. Two Body motion – the two end bodies are in orbit around a spherical homogenous earth, and the gravitational field is the field of the central body. In addition,the two bodies are assumed to be small enough that there is no mutual attraction.

2. Orbital Motion – for simplification, orbital motion of the two bodies is constrained to remain in the orbit plane. The reason is that the tension forces from the tether will remain almost entirely within the plane of the initial orbit and there are no other perturbing forces acting out of the plane.

3. Tether Tension – this will be the control parameter for the optimization process. For this reason, the tension will be modeled as an N–dimensional polynomial of the form

$$F_T = \Sigma\, C_i\, D^i$$

where F_T is the magnitude of the tension, D is the distance between the ET and the Shuttle, and the C_i's

are the optimized coefficients that maximize the altitude gain for the shuttle. In addition, the tension generated in the cable during the tether "toss" will be treated as a generalized force rather than as being generated by a potential similar to the gravitational energy.

4. Tether Length – to avoid problems of overstressing available material limits, the tether length will be limited to a maximum of 100 kilometers. In addition, although many materials are suitable for use this project will use Kevlar as the material of choice for the cable.

5. Air Drag – the force of air drag will be included in the physical model of the tether system and it will be modeled as being of the of the general form

$$F_D = 1/2\ C_D\ A\ \rho\ V^2$$

where the atmospheric density, ρ , is determined from an exponential atmosphere model. The coefficient of drag, C_D , and the cross–sectional area, A , of the bodies are averages for the external tank and the shuttle. The model will ignore the small effect of drag on the tether since it has a very small cross sectional area normal to the velocity. Since the

separation point of the shuttle from the tank is at the normal apogee of the tank's orbit, a further simplification of the drag problem is that the shuttle will be assumed to be above the atmosphere for the portion of the flight which involves the use of the tether, and the air drag forces will only be considered as acting on the external tank.

The equations describing the motion of the system are derived using the classical Lagrangian Procedure. The tension force in the tether is represented as a generalized force, and serves as the control used during the optimization process to maximize the altitude gain by the Shuttle during a "toss" from the External Tank. The determination of the air drag forces that act on the system are included under the generalized forces acting on the system. The derivation of the equations of motion for this dumbbell shaped two body mass in the next section describes in more detail the above listed assumptions, and later sections of this report will further explain the optimization concept used.

II. Derivation of Equations of Motion

Consider a physical system of the Space Shuttle and the External Tank having masses m_s and m_t, respectively, which are connected by a tether that has mass density, ρ_t, and the entire system is in orbit around a perfectly spherical and homogeneous Earth. In the most general case, the orbit of the two body system around the Earth is Keplerian and the two end bodies may be represented as idealized point masses with relative position vectors in a δr, $\delta\theta$ local co-ordinate frame centered around a mean circular orbit of radius $\underline{R}_{ref}$. The local coordinate frame is defined with the δr axis pointing outward along a line from the origin of the geocentric frame to the location of mean circular reference point (this is aligned with the vector $\underline{R}_{ref}$), and with the $\delta\theta$ axis at a right angle to this in the direction of the orbital velocity and in the plane of the orbit. Both of these unit vector directions are in the orbit plane, and all motion is assumed to remain in the plane of the orbit. The two end bodies will be located by the vectors $\underline{R}_{sref}$ for the shuttle, and $\underline{R}_{tref}$ for the external tank both of which are measured in the relative reference frame. Since the typical separation

distances between the tank and shuttle are measured in kilometers, the tether's cross section (measured in centimeters, or fractions thereof) is very small when compared to it's length and so bending and torsional stiffness forces resulting from the diameter of the tether cable will be very small, and the effects of those forces may be neglected since they contribute little to the dynamics of the system. For the purpose of evaluating the motion of the shuttle and external tank tether connected system, the contributions of the tether mass with respect to the system's potential and kinetic energies will be accounted for by adding part of the tether's mass to each of the end bodies. This will reduce the three component system (shuttle, external tank, and tether) to a lumped two point mass model simplifying the tethered system dynamics without reducing the accuracy of the solution. The development of the tethered system equations of motion will be done using the classical Lagrangian development method *summing* the contributions of each individual part of the system to the total kinetic and potential energies. The Lagrangian development of the equations of motion for this two body tethered system used in this determination of the optimum tether tension profile rest on the following assumptions:

(1) the geocentric equatorial reference frame is an inertial one;

(2) the earth's gravitational field is uniform, and the center of the attracting field is the center of the geocentric co–ordinate frame;

(3) the earth is a perfectly spherical body;

(4) the atmospheric density of the earth's atmosphere, ρ_a , is dependent only on the altitude, h , above the surface of the Earth, and is related to a nominal value, ρ_{ref} , by an exponential relationship;

(5) the atmosphere rotates with the Earth like a rigid body about the Z axis of the geocentric co–ordinate frame; and

(6) the relative motion of the end bodies remains in the orbital plane, which is initially defined by the positions and velocities of the shuttle and external tank just prior to initial separation.

This model assumes that the tether mass is distributed uniformly along the straight line between the location of the two end body point masses, i.e. – along the vector $\underline{R} = \underline{R}_{orel} - \underline{R}_{trel}$ as in Figure 2. From this, it is obvious that the tether must have a uniform mass density, ρ_t , throughout it's length and the cross sectional area is assumed to be constant. In addition, the tether is assumed to have uniform deformation along the line between the two end bodies within the material limitations of the tether.

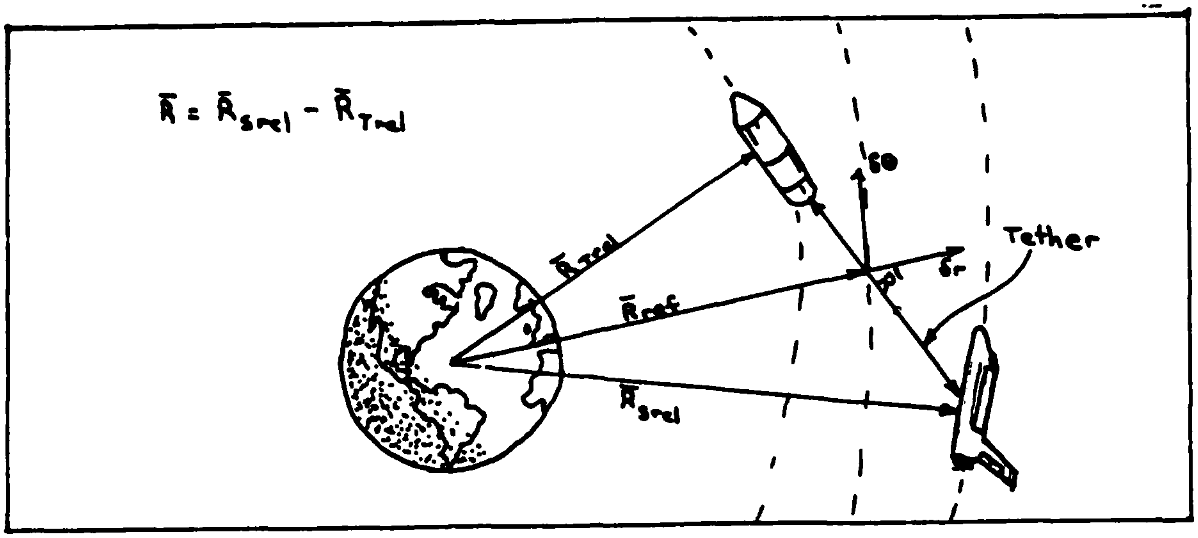

Figure 2 Tether Position along vector $\underline{\mathbf{R}}$

The simplification of the system dynamics as outlined in these assumptions is realistic as long as the tether lies along the line between the two bodies, but if the tether does not there may be significant deviations from this idealized model of the mass distributions. Nevertheless, the kinetic and potential energies that are calculated from these assumptions should be very representative of the actual values as a mean.

The attitude motion of the system out of the orbit plane will have very little effect on the orbital motion. Hence the relative orbital equations of motion of each piece can be determined using the classical Lagrangian formulation for the orbital motion of a point mass. The Lagrangian of a system is determined from the sum of the kinetic and potential energies of the system's parts. The kinetic energy ($\mathcal{T}$) of this system is determined by the motion of the two point masses as follows:

$$\mathcal{T} = 1/2\ m_s\ V_{srel}^2 + 1/2\ m_t\ V_{trel}^2 \tag{1}$$

Where m_s is the mass of the space shuttle, and V_{srel} is the magnitude of the inertial velocity of the shuttle expressed in the local δr , $\delta\theta$ relative coordinate system. Similarly, m_t and V_{trel} are the mass and relative velocity of the external tank in the relative coordinate system. However, the velocity terms, V_{srel} and V_{trel} , in the above expression may be expressed as the time derivative of their respective position vectors (i.e. – $\underline{V} = d\underline{R}/dt$), and this expression for the velocity will be substituted in the kinetic energy expression. The potential energy of the system consists of the sum of the gravitational potential energy of the two end bodies, and may be represented as:

$$\mathcal{V}_{TOT} = \mathcal{V}_s + \mathcal{V}_t \tag{2}$$

Where $\mathcal{V}_s$ represents the potential energy of the shuttle, and $\mathcal{V}_t$ represents the potential energy of the external tank. Thus, using the assumptions listed previously for the definition of the coordinate frames and assuming a spherical homogeneous Earth with a uniform gravitational field, the gravitational potential energy of the two parts of the system may be expressed as follows:

$$\mathcal{V}_s = -\mu_e \, m_s \, / \, (\, R_{ref} + \delta r_s \,) \tag{3}$$

and

$$\mathcal{V}_t = -\mu_e \, m_t \, / \, (\, R_{ref} + \delta r_t \,) \tag{4}$$

Where μ_e is the earth's gravitational parameter, and the terms $(R_{ref} + \delta r_s)$ and $(R_{ref} + \delta r_t)$ are the distance of the shuttle and external tank in the radial direction from the center of the geocentric coordinate frame. Thus, the total potential energy of the system may be expressed as:

$$\mathcal{V}_{TOT} = [- \mu_e \, (\, m_t \, / \, (\, R_{ref} + \delta r_t \,) + m_s \, / \, (\, R_{ref} + \delta r_s \,))] \tag{5}$$

Then combining the expressions for the potential and kinetic energies, the Lagrangian for this system is:

$$\begin{aligned} \mathcal{L} \; = \; & 1/2 \, m_s \, V_s^2 \; + \; 1/2 \, m_t \, V_t^2 \\ & + \mu_e \, [(\, m_t \, / \, (\, R_{ref} + \delta r_t \,) + m_s \, / \, (\, R_{ref} + \delta r_s \,) \,)] \end{aligned} \tag{6}$$

Where the magnitudes of the velocities in the above equation may be expressed in terms of their components as follows:

$$V_s = [(d\,\delta r_s /dt)^2 + (\Omega + d\,\delta\theta_s /dt)^2 (R_{ref} + \delta r_s)^2]^{1/2} \tag{7}$$

and

$$V_t = [(d\,\delta r_t /dt)^2 + (\Omega + d\,\delta\theta_t /dt)^2 (R_{ref} + \delta r_t)^2]^{1/2} \tag{8}$$

In the above expressions, Ω is the angular velocity of the mean circular orbit used as a reference point, and R_{ref} is the magnitude of the radius vector of the mean circular orbit expressed in the geocentric coordinate frame. Using these expressions for the velocity, the expression for the Lagrangian may be expressed in terms of the components of the position and velocity vectors. Thus, making these substitutions the complete expression for the system's Lagrangian is expressed as:

$$\begin{aligned}\mathscr{L} = {} & 1/2\, m_s [(d\,\delta r_s /dt)^2 + (\Omega + d\,\delta\theta_s /dt)^2 (R_{ref} + \delta r_s)^2] \\ & + 1/2\, m_t [(d\,\delta r_t /dt)^2 + (\Omega + d\,\delta\theta_t /dt)^2 (R_{ref} + \delta r_t)^2] \\ & + \mu_e [(m_t / (R_{ref} + \delta r_t) + m_s / (R_{ref} + \delta r_s))] \end{aligned} \tag{9}$$

The external generalized forces that will be acting on the system will be limited to two, those of air drag and the tension force of the tether. The tension in the tether will be considered under the heading of the generalized forces because the tension profile used during the reel–in maneuver process is the control variable used to optimize the altitude gain of the space shuttle.

For modeling purposes in this report, the tension force in the tether will be represented as an Nth degree polynomial of the form:

$$F_T = C_1D + C_2D^2 + C_3D^3 + C_4D^4 \,.\,.\,.\,.\,. + C_iD^i \qquad (10)$$

or in short hand notation,

$$F_T = \Sigma\ C_i\ D^i \qquad (i=1,n) \qquad (11)$$

The following assumptions on the tether's behavior were made in this simulation to maintain a simple model:

(1) The resultant force from the strain energy of the tether acts only in the axial direction due to the small cross section of the tether, and is negligible compared to other forces acting on the system;

(2) The tether has a uniform mass density per unit length, ρ_t , and a constant cross sectional area throughout it's length;

(3) The tether lies along the straight line connecting the point mass representations of the shuttle and the external tank;

(4) There is uniform deformation along the tether within the material limits of the tether.

These simplifications for the tether are sufficiently realistic when the tether is stretched between the two bodies. For the time periods when the tether is slack, there may be significant deviations from the assumed mass distribution. However, the answers that are generated should still be fairly representative of the mean values for the forces involved.

The force of air drag will be very small for objects at the 200+ km altitude of the shuttle's planned orbit when compared to the attraction of gravity and the force of tension from the tether. Thus, air drag in the model will be considered as acting only on the external tank with it's lower orbit causing the tank to spend more of it's time in the atmosphere. In general, the force of air drag on the tank may be represented as:

$$\mathbf{F}_D = 1/2\ C_D\ A\ \rho\ V_{Trel}^{\ 2} \tag{12}$$

Where the variable C_D is the coefficient of drag (averaged) for the tank, $\mathbf{A}$ is the cross sectional area of the tank, ρ is the atmospheric

density, and V_{Trel} is the magnitude of the inertial velocity of the tank expressed in the relative frame. The force of air drag is parallel to the velocity vector, $\underline{V}_{Trel}$, but in the opposite direction thus opposing the motion of the external tank. The magnitude of the velocity, V_{Trel} , is measured with reference to the geocentric frame. Since the density of the atmosphere can vary considerably with altitude, sun spot activity, and many other factors a simple method of modeling this variable was chosen. One of the simplest models for determining the air density is an exponential model using a scale height, and reference value for the air density. For this type of model, the variations in the density, ρ , is calculated from the formula:

$$\rho = \rho_{ref} \exp^{-(h / href)} \tag{13}$$

The reference density, ρ_{ref} , is determined for conditions at the reference altitude, and in this case the value used was 1.2321×10^{-10} kg/m^3 (20:192). The scale height, **href** , is the altitude used that gives the best density match for the altitude range in question. The value used was for this program was **href** $= 2 \times 10^5$ meters (20:192). The height, h , of the external tank is determined by $h = R_T - R_E$, with both vectors being expressed in the geocentric coordinate frame.

In general, the equations of motion are determined using the Lagrangian formulation are of the form:

$$d/dt(\partial\mathcal{L}/\partial\dot{q}_i) - (\partial\mathcal{L}/\partial q_i) - Q_i = 0 \qquad (14)$$

where $\mathcal{L}$ is the Lagrangian for the system, q_i and $\dot{q}_i$ are the generalized Lagrangian coordinates and their first derivatives, and the Q_i's are the generalized forces acting on the system for that generalized coordinate direction. For this two body tethered system, the generalized Lagrangian coordinates are as follows:

q_i's -- δr_s	$\dot{q}_i$'s -- $d\,\delta r_s\,/dt$
$\delta\theta_s$	$d\,\delta\theta_s\,/dt$
δr_t	$d\,\delta r_t\,/dt$
$\delta\theta_t$	$d\,\delta\theta_t\,/dt$

The generalized forces acting on the system are air drag forces on the external tank, and the tether force acting on the external tank and the shuttle. Using the classical Lagrange method of determining the generalized forces for each of the generalized coordinate variables, they may be expressed as:

$$Q_k = \Sigma\ \mathbf{F}_j\ \partial \underline{r}_k/\partial q_j \qquad (j=1,p \text{ and } k=1,n) \qquad (15)$$

Where the generalized force in the equation is expressed as Q_k , the forces acting on the system as $\mathbf{F}_j$, p is the number of forces acting on the system, and n is the number of generalized coordinates. Since the forces acting on the system are air drag and the tension force of the tether, the result of evaluating equation 15 for each of the coordinate directions is as follows:

$$Q_1 = (\mathbf{F}_D + \mathbf{F}_T)_{drshut} \tag{16}$$

$$Q_2 = (\mathbf{F}_D + \mathbf{F}_T)_{dthetashut} \tag{17}$$

$$Q_3 = (\mathbf{F}_D + \mathbf{F}_T)_{drtank} \tag{18}$$

$$Q_4 = (\mathbf{F}_D + \mathbf{F}_T)_{dthetatank} \tag{19}$$

Where the respective Q_i's are for the forces on the shuttle in the δr direction (Q_1), the forces on the shuttle in the $\delta\theta$ direction (Q_2), the forces on the tank in the δr direction (Q_3), and the forces on the tank in the $\delta\theta$ direction (Q_4). However, since the air drag term is negligibly small on the shuttle, $\mathbf{F}_D$ may be assumed to be zero on equations 16 and 17 thus further simplifying the equations of motion. Then using the definitions for the Lagrangian coordinates, the general definition of the equations of

motion from Eq. 14, and the generalized force definitions from Equations 16 through 19 it is a simple matter to express the equations of motion as:

For $\delta \mathbf{r}_s$

$$m_s\ d^2\,\delta\mathbf{r}_s\,/dt^2 - m_s(\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ )(\ \Omega + d\ \delta\theta_s\ /dt\ )^2 +$$
$$(\ \mu_e\ m_s\ )\ /\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ )^2 - F_{Trs} = 0 \qquad (20)$$

For $\delta\theta_s$

$$2\ \Omega\ \ d\ \delta\mathbf{r}_s\ /dt\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ ) + d^2\ \delta\theta_s\ /dt^2\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ )^2$$
$$+\ 2\ d\ \delta\theta_s\ /dt\ \ d\ \delta\mathbf{r}_s\ /dt\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ )$$
$$-\ F_{Tstheta}\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_s\ )\ /\ m_s = 0 \qquad (21)$$

For $\delta\mathbf{r}_t$

$$m_t\ d^2\,\delta\mathbf{r}_t\,/dt^2 - m_t(\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ )(\ \Omega + d\ \delta\theta_t\ /dt\ )^2 +$$
$$(\ \mu_e\ m_t\ )\ /\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ )^2 - F_{Trtank} - F_{Drtank} = 0 \qquad (22)$$

For $\delta\theta_t$

$$2\ \Omega\ \ d\ \delta\mathbf{r}_t\ /dt\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ ) + d^2\ \delta\theta_t\ /dt^2\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ )^2$$
$$+\ \ 2\ d\ \delta\theta_t\ /dt\ \ d\ \delta\mathbf{r}_t\ /dt\ (\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ )$$
$$-\ (\ F_{Tthetatank} + F_{Dthetatank}\ )(\ \mathbf{R}_{ref} + \delta\mathbf{r}_t\ )\ /\ m_t = 0 \qquad (23)$$

By expanding the gravity potential terms of equations 20 through 23 using the binomial theorem, it is possible to eliminate the gravitational parameter, μ_e , from the equations. During the expansion, in order to linearize the equations of motion the second order terms that contained δr and $\delta\theta$ were ignored as they were many magnitudes smaller than the terms containing R_{ref}. In addition, the terms that were of the form δr / R_{ref} and $\delta\theta$ / R_{ref} were ignored since the result was more than three orders of magnitude less than many of the other terms. Since the ultimate objective is to determine the motion of the two end bodies as a function of time, the accelerations in each of the coordinate directions must be determined and then integrated twice to yield the position vectors. Thus simplifying the equations further by combining terms and dropping the higher order terms involving δr and $\delta\theta$, it is possible to solve equations 20 through 23 for the accelerations along the δr and $\delta\theta$ component axis. When the forces of air drag and tension are broken down into their components along the relative coordinate axis, the resulting set of equations is:

For δr_s

$$d^2 \delta r_s /dt^2 = 3 \delta r_s \Omega^2 + 2 R_{ref}\, d \delta\theta_s /dt \ \Omega + F_{Trs}/m_s \tag{24}$$

For $\delta\theta_s$

$$d^2 \delta\theta_s /dt^2 = - (2 \Omega\, d \delta r_s /dt) / R_{ref} + F_{Ttheta} / (R_{ref}\, m_s) - (\delta r_s F_{Tthetas}) / (R_{ref}^2\, m_s) \tag{25}$$

For δr_t

$$d^2 \delta r_t /dt^2 = 3 \delta r_t \Omega^2 + 2 R_{ref} d \delta\theta_t /dt \Omega + (F_{Trtank} + F_{Drtank}) / m_t \quad (26)$$

For $\delta\theta_t$

$$d^2 \delta\theta_t /dt^2 = - (2 \Omega d \delta r_t /dt) / R_{ref} + (F_{Ttheta} + F_{Dtheta}) / (R_{ref} m_t) - (\delta r_t(F_{Tthetatank} + F_{Dthetatank})) / (R_{ref}^2 m_t) \quad (27)$$

Then, using the above expressions for the accelerations in each local coordinate direction, determining the motion of the two bodies will require the double integration of each of these equations (24 to 27) of motion. With the linearized expressions for the acceleration described in equations 24 to 27, it is possible to solve for the motion of the two end bodies using a state space representation of the dynamics. For this process to work in this program let $X_1 = \delta r_s$, $X_2 = \delta\theta_s$, $X_3 = d \delta r_s /dt$, $X_4 = d \delta\theta_s /dt$, $X_5 = \delta r_t$, $X_6 = \delta\theta_t$, $X_7 = d \delta r_t /dt$, and $X_8 = d \delta\theta_t /dt$. Using this representation, the equations of motion are consolidated into a set of equations and integrating the system of linearized equations with a numerical

integrator, such as Haming, is relatively easy to accomplish. The determination of the tether force from it's idealized coefficients is the only input to this set of equations of motion that is subject to control external to the basic physics of the system. Thus, the current choice of the N coefficients will determine the altitude gain the shuttle achieves for this set of parameters. The determination of what are the best choice for the optimized coefficients, and what is the most efficient method of finding them is an optimization problem of great difficulty. The next chapter will develop in detail the method of maximizing the altitude gain of the space shuttle by correctly choosing these coefficients.

III. Development of the Optimization Routine

The purpose of this research is to develop a method for maximizing the altitude gain of the Space Shuttle from a tethered exchange of momentum with the External Tank. The altitude gained may be determined by arbitrarily picking a nominal set of coefficients, determining the apogee of that orbit (and hence the altitude) by numerically integrating the orbit from the point of tether release, then using the computer driven optimization routine to incrementally improve the choice of coefficients and then re-integrating the "new" orbit to determine the altitude gain from the "better" guess. However, it would be much simpler to be able to determine the performance of the program by comparing the change from the reference orbit to the new orbit by the change in classical orbital elements a , e , i , and $\mathcal{E}$. It is much more computationally effective to determine the altitude gain by determination of the orbital energy, $\mathcal{E}$, from the radius and velocity vectors, $\underline{R}_s$ and $\underline{V}_s$, at the time of tether detachment than numerically integrate the entire post-release orbit to determine the maximum altitude.

Since altitude is related to the orbital radius, an equally effective alternative to altitude, h , might be the semi–major axis, a . This increase in a is easy to measure, and allows a quick determination of the performance of the tether "toss". However, the added momentum from such a "toss" would show immediately in the orbital energy of the shuttle, $\mathcal{E}_s$, since it measures the kinetic and potential energy of the shuttle at any point and is a constant for a closed orbit (i.e. – a circle or an ellipse) with no dissipative forces involved. Since the classical parameters require less computation time, the change in the energy, $\mathcal{E}_s$, will be used to determine the performance of the tether maneuver. The shuttle's orbital energy, $\mathcal{E}_s$, is related to the semi–major axis, a , of the shuttle's orbit by the formula:

$$a = -\left(\mu_e / \left(2\,\mathcal{E}_s \right)\right) \tag{28}$$

This variable can be directly related to the altitude gain of the maneuver since $a = (R_a + R_p)/2$, where R_a is the radius of the orbit at apogee and R_p is the radius of the orbit at perigee. Thus, if the orbital energy was increased, the semi–major axis (and orbital altitude) would also be increased. As a review of what the magnitude of orbital energy relates to, for a parabolic orbit the orbital energy is equal to zero, and the energy of a hyperbolic orbit is greater than zero. Since the shuttle is equipped for earth orbit, closed form orbits (ellipse or circle) are the only ones of interest, and for these orbits the orbital energy, $\mathcal{E}_s$, is always negative. Thus, the maximum altitude gain (in a closed orbit) for a

momentum "toss" would be when the orbital energy is increased to just less than zero. However, if the increase in altitude is on the order of 100 kilometers, the increase may well be worth the extra complexity and weight of the added tether equipment.

Since the only control variable in the development of the equations of motion in Section II was the tension force in the tether, the orbital energy may be expressed as a function of the tether tension coefficients, or

$$\mathcal{E}_s = \mathcal{F}(C_1, C_2, C_3, \ldots\ldots C_n) \tag{29}$$

Then expanding the expression for the orbital energy in equation 29 by using a second order Taylor series approximation allows the expression of the shuttle's orbital energy as follows:

$$\mathcal{E}_s = \mathcal{E}_{s0} + \Sigma [\partial\mathcal{E}_s/\partial C_i \ \delta C_i \ + \ 1/2 \ \ \delta C_i^T \ \ \partial^2\mathcal{E}_s/\partial C_i^2 \ \ \delta C_i]_{\ (i=1,n)} \tag{30}$$

Where $\mathcal{E}_{s0}$ is the value of the orbital energy for some nominal set of tether tension coefficients, n is the number of coefficients used in the tether tension polynomial, δC_i is the change made to the coefficient for this perturbation, and $\partial\mathcal{E}_s/\partial C_i$ and $\partial^2\mathcal{E}_s/\partial C_i^2$ are the first and second derivatives of the orbital energy with respect to the coefficient being perturbed.

Since this equation is a summation over the number of coefficients being used (variable n) to model the tether tension, it is more convenient to re–write equation 29 in a matrix format that allows the expression to represent the entire set of variables being used. If we let $\underline{\mathbf{X}}$ be the column matrix of tether tension coefficients, C_i's, then the summation of the first and second derivatives of the orbital energy per change in the coefficients may be expressed as $\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}$ and $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2$. These partial derivatives are evaluated at a set of conditions picked arbitrarily at the start of the optimization process. Then, using the matrix definition of $\underline{\mathbf{X}}$ for the tension coefficients, $\delta\underline{\mathbf{X}}$ would be the small perturbations of the coefficients around the nominal value used for the analysis. With these definitions, the second order Taylor expansion of the orbital energy, $\mathcal{E}_s$, from equation 30 may be expressed in matrix form as:

$$\mathcal{E}_s = \mathcal{E}_{s\ nom} + \partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{nom}\ \delta\underline{\mathbf{X}} + 1/2\ \delta\underline{\mathbf{X}}^T\ \partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{nom}\ \delta\underline{\mathbf{X}} \quad (31)$$

Thus with a method of relating the orbital energy (and hence the altitude) to the change in coefficients, the question now becomes how to chose the optimum set of coefficients. In order to determine if the current values for the coefficients in the expression for the energy actually establish the maximum value, the classical tests for determination of maximums and minimums are adapted to matrix form. If the current set of coefficients is a maximum, the first derivative, $\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{nom}$, should be equal to zero, and the sign of the second derivative, $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{nom}$, should be negative. If

the evaluation of the expressions for the first and second derivatives shows that the current choice of coefficients does not yield a maximum value for the orbital energy, then an improved guess must be determined for the coefficient matrix $\underline{\mathbf{X}}$. This is done by taking the derivative of equation 28 with respect to the coefficient matrix, $\underline{\mathbf{X}}$, and setting the resulting equation equal to zero. Taking the derivatives of the matrices in equation 31 with respect to the coefficient matrix yields:

$$\partial\mathcal{E}_s/\partial\underline{\mathbf{X}} = \partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{\,\mathrm{nom}} + \partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,\mathrm{nom}}\ \delta\underline{\mathbf{X}} \tag{32}$$

Then setting equation 32 equal to zero would yield the following:

$$\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{\,\mathrm{nom}} + \partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,\mathrm{nom}}\ \delta\underline{\mathbf{X}} = 0 \tag{33}$$

Simplifying this equation, and solving for the first derivative of the orbital energy leads to:

$$-\ \partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{\,\mathrm{nom}} = \partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,\mathrm{nom}}\ \delta\underline{\mathbf{X}} \tag{34}$$

Since the chances of picking the correct coefficients to generate the maximum energy for the shuttle on the first iteration are exceedingly small, to determine the best correction for our present guess, $\underline{\mathbf{X}}^i$, we must solve equation 34 for the term $\delta\underline{\mathbf{X}}$. To do this requires a method of solving n linear equations (one for each of the coefficients), which is expressed in matrix form as $\mathbf{A}\ \mathbf{x} = \mathbf{B}$, where the matrix we want to determine from this expression is x. Equating this general form to equation 34, $\mathbf{A}$ is equivalent to the matrix $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,nom}$, $\mathbf{x}$ is the same as matrix $\delta\underline{\mathbf{X}}$, and $\mathbf{B}$ is the matrix $-\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{\,nom}$. While the determination of the matrix might be made by finding the inverse of $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,nom}$, it is not advisable to solve the equation this way because $[\ \partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,nom}\ ]^{-1}$ might be singular. Thus to determine the answer simultaneously to the N linear equations will require a flexible linear equation solver that is capable of obtaining precise answers from ill–conditioned matrices. For this project, the process used was the VAX IMSL routine **LEQT2F**. The routine inputs the current values for the matrices $\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{\,nom}$ and $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{\,nom}$ and then uses a gauss–seidel method with partial pivoting to determine the $\delta\underline{\mathbf{X}}$ matrix.

Upon determining $\delta\underline{\mathbf{X}}$, the incremental improvement to the coefficients, the next value to be used in the iteration process can be determined by adding the correction to the present value of $\underline{\mathbf{X}}$ as expressed by:

$$\underline{\mathbf{X}}^{i+1} = \underline{\mathbf{X}}^i + \delta\underline{\mathbf{X}}^i \tag{35}$$

The new value, $\underline{X}^{i+1}$, for the matrix of coefficients is input into the equation of motion routine to determine the resultant boost that the tether "toss" gives to the shuttle. The iterative process of determining the next value for $\delta\underline{X}^i$ continues until a maximum value for $\mathcal{E}_s$ is reached.

When the Lagrangian for the system is non–linear and the control variable is linear then it is possible to simplify the optimization process further. By examining the results of equation 9, it is easy to determine that the system Lagrangian is non–linear. While the Lagrangian for this system is not linear, the equations of motion that were derived from it are linear because of the assumptions and simplifications made during their derivation. In addition, the control variable for this optimization problem is the tension in the tether, and it is linear. According to Bryson and Ho (10:252) , when this type of system exists there is only one method of solution and it is called a <u>Bang – Singular – Bang</u> solution. This <u>Bang – Singular – Bang</u> type of system has it's optimum performance when the control variable is turned off until a certain point and then once turned on the control variable operates at it's maximum value to drive the system performance. The conditions for this system that equate to this type of control method may be expressed as:

$$F_T = F_{T\max} \tag{36a}$$

where F_{Tmax} is determined by the characteristics of the material used for the tether. The other option for the value of the tension force is

$$F_T = 0.0 \tag{36b}$$

Where the maximum tension, F_{Tmax} , is determined by the material properties of the cable that is being used, and the physical size of the cable. For this project, the material selected was Kevlar and the nominal diameter of the cable was 1.0 centimeter. Since the maximum tension in the cable is determined by the product of Young's Modulus for the material, E, and the cross sectional area of the cable, A, then the maximum yield strength of this tether is 2.120575 x 10^6 Newtons. Using the <u>Bang – Singular – Bang</u> type of optimal control, with the cable tension fixed at F_{Tmax} , the only remaining variable not analytically determined is the initial separation distance, D_0 , at which the tension force is applied to the cable. The variation of the initial separation distance, D_0 , will make a difference in the final altitude that the shuttle will reach because this distance determines the length of time that the cable will transfer momentum from the external tank to the shuttle. Using the same method for determining the optimum energy value as was developed for determining the solution for the tension coefficients, it is obvious that the orbital energy, $\mathcal{E}_s$, is also a function of the initial distance. Using the same form of expression as in equation 29, it would be possible to write a Taylor series expansion of the orbital energy in terms of the separation distance, D . However, the end result would be the

same expression as equation 30 and solving this equation for the resulting $\delta\underline{X}$ matrix would require the same solution techniques as for equation 34. If $\delta\underline{X}$ is now defined as the 1 x 1 matrix (a scalar) containing the separation distance, D , it is possible to use equation 34 to solve for the optimum value. Thus, the solution to the incremental adjustment process is still determined by the solution to equation 34. The only difference in the process as it was described above is that instead of N linear equations to solve (one for each coefficient) there is only the one equation for the value of the initial separation distance. With the initial separation distance as the variable in the $\delta\underline{X}$ matrix, the matrix is now only a scalar and a complex equation solver is not required as with the N tension coefficients.

Theoretical analysis of the system equations of motion enabled a further simplification to be made in the optimization process by eliminating the complex problem of having to determine the polynomial representation of the tether tension. Due to the nature of the three body system, the optimal control solution to maximizing the altitude of the shuttle is to have the tension in the tether be equal to the material limits of the cable, or have no tension in the tether at all. This simple <u>Bang – Singular – Bang</u> system means that the only variable once the size of the cable has been chosen is when, measured in terms of initial separation distance, to "turn on" the cable tension.

IV. Program Development and Method of Solution

The theory presented above for the perturbation process determining the optimum value for the separation distance at which to appl the tension force on the cable has been programmed in FORTRAN. An o ine of the computer algorithm will facilitate understanding of the results t are presented in the next section. The physical values for the size d weight of the external tank and shuttle are part of the input routine, as ll as the initial value for the separation distance. The initial conditions the end body position and velocity are input as data in the geocentric e atorial coordinate frame, and after program initiation they are transfor d into the local relative coordinate frame described in Section II. With th relative initial conditions for position and velocity, the process still requ s a source for choosing the "turn on" distance of the tension in the tether Any arbitrary value for the separation distance may be selected, and he equations of motion for the end bodies (equations 24 to 27 in Section II are numerically integrated in the relative frame using the numerical itegrator **Haming**. The tension force is held equal to zero during the ca lation of the trajectories of the two end bodies until the distance betwee hem has

exceeded the selected value for the separation distance. Once the tether force has been "turned on", it's input in the numerical integration process continues until the separation between the bodies has reached the maximum tether length. There are traps in the program to avoid pulling on the tank for momentum transfer after it has crashed on the earth, if the tension in the tether becomes negative, or if it is in a higher orbit than the shuttle. When the numerical integration of the orbit has been completed, the position and velocity of the shuttle at t_f, the end time, are transformed from the local reference frame into the geocentric equatorial frame so that the classical orbital elements of the post–release orbit for the shuttle may be determined. Then the result of that separation distance can be judged by comparing the orbital energy after the "toss" with the nominal orbital energy, $\mathcal{E}_{nom}$, and any other values to determine a maximum. This basic outline is what the program does for each separation value that is input, and the first cycle of the equation of motion subroutine is to determine the nominal energy, $\mathcal{E}_{nom}$, value at the reference point.

After a nominal value for the energy is determined, a perturbation process is used to explore the sensitivity of the energy to changes in the separation distance. In the perturbation process, each iteration determines five perturbed values that cluster around the nominal value. These perturbed values for the distance are used to analyze what direction and magnitude of change to use for the next improvement to the separation distance. Using the process described above for determining $\mathcal{E}_{nom}$, the integration is repeated for each of the perturbed separation distances. Then the position and velocity vectors are transformed and the orbital energy is determined at each of these points. By then using numerical analysis

techniques to analyze the five points (1:883–885), it is possible to find values for the first, $\partial\mathcal{E}_s/\partial\underline{\mathbf{X}}\,|_{nom}$, and second derivatives, $\partial^2\mathcal{E}_s/\partial\underline{\mathbf{X}}^2\,|_{nom}$, of the orbital energy at that nominal separation distance. The tests described in Section III for a maximum are run on these two derivatives, and if necessary the next improvement to the separation distance, $\delta\underline{\mathbf{X}}$, is determined. Through the equations for determining $\delta\underline{\mathbf{X}}$ listed in Section III (equations 31 to 35), the next incremental improvement to the $\underline{\mathbf{X}}$ matrix is determined. With the new value for the separation distance from equation 35, the iterative process is repeated until the separation distance for the maximum value of orbital energy is reached. In this process, it is important to remember that the orbital energy of all orbits under consideration have a negative value ($\mathcal{E}$ <= 0.00), and to maximize this value will be to make it's value marginally less than zero.

V. Results and Discussion

The validity of the equations of motion developed in Section II for the simplified two body system of the shuttle and the external tank has been validated by comparing the result of integrating the equations of motion with the classical orbital elements of a known reference orbit. To further check the validity of the computer implementation of the equations of motion, the position vectors of the two end bodies were input and after transformation to the local frame, integrated for 50,000 seconds in simple two body motion with no external forces, such as air drag, to compute the position vectors. Then the local coordinate frame position vectors were transformed back into the geocentric frame, and the classical orbital elements were calculated to compare with those of the two bodies at the initial time. The small changes in orbital elements from the initial time to the final time can be accounted for by round–off error in the transformation from inertial geocentric coordinates to the local relative frame, and then back again. For example, the difference in a , the semi–major axis, over the trial period was less than .5 meters with the initial orbital radius being about 6,444,000 meters.

Since this is a change of approximately 7.76 x 10^{-06} per cent, it is obvious that the simple two body portion of the model is operating correctly.

For the case that was investigated in this paper, the shuttle and external tank are assumed to be at the normal separation point and the shuttle has started to pay out the cable as it moves away from the external tank. The separation point is at an altitude of 65 kilometers, and the shuttle has already acquired the velocity to carry it to it's normal orbital height. From the initial run of the program, these initial conditions give the shuttle a semi–major axis of it's orbit of about 6490 kilometers which is used as the absolute standard to judge performance of the tether "toss".

While previous investigations (12:167) into the use of the tether as a means to augment rockets for propulsion have shown that the boost available from a tether is proportional to the length of the tether, they assumed a constant tension in the tether from the time it was deployed until the tether was discarded. The method of this research, varying the point at which tension is applied to a constant tether length, has the same effect as changing the length of cable used on a tether "toss". For the 1.00 centimeter tether used in the Bang – Singular – Bang model that was described in Section III, the results of variations in the orbital energy, $\mathcal{E}_s$, for the different initial separation distances of the post release orbit are shown in **Figure 3**. The graph in **Figure 3** measures the initial separation distance at which the tension is applied (in kilometers) on the horizontal axis, and the resulting orbital energy from the "toss" along the vertical axis. The orbital energy is a measure of the potential and kinetic energy of the shuttle, and by using dimensional analysis it is easy to determine that the units used for this term are $(\text{length})^2/(\text{time})^2$, and this is the specific

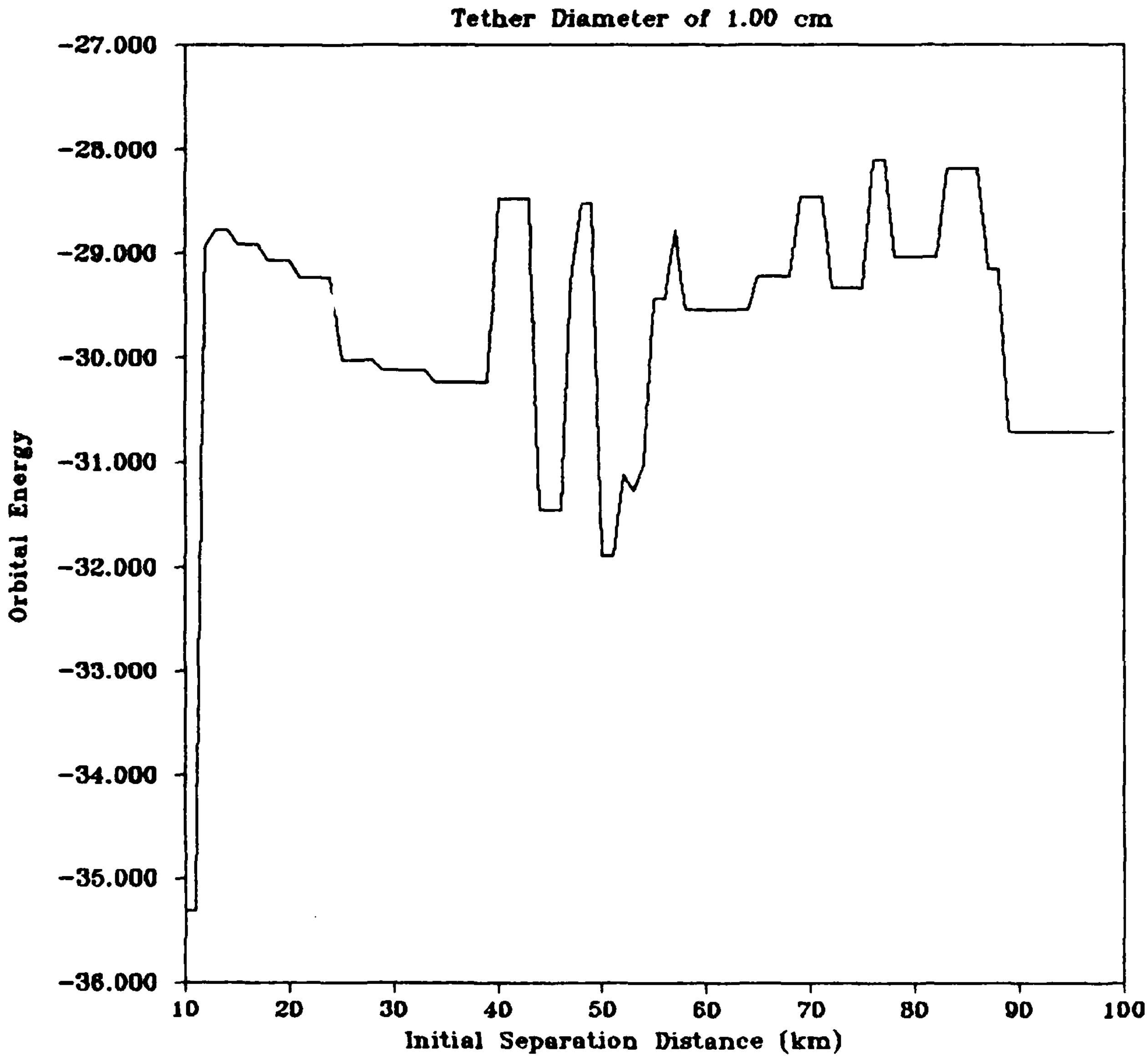

Figure 3 Initial Separation versus Orbital Energy

mechanical energy of the shuttle. For this model, the velocities of the bodies were measured in km/sec and their masses were measured in kilograms, thus $\mathcal{E}_s$ is measured in km²/sec².

Since the orbital energy term, $\mathcal{E}_s$, is somewhat hard to relate to the actual performance of the "toss" on the shuttle, using Equation 28 it is possible to determine **a** , the semi–major axis, for the various separation distances shown in **Figure 3**. Then plotting these values of the semi–major axis, **a** , for the same range of initial separation distances as was shown in **Figure 3** will yield the graph that is shown in **Figure 4**.

The smallest distance that is on the graph of **Figures 3** and **4** is 10 kilometers because for distances less than this the time required for the tank and shuttle to reach the 100 kilometer length of the tether exceeds 15 hours, and this is too long to be of use. In addition, the resultant "toss" from the tether actually ends up pulling the shuttle downwards because the tank is still very close and the resultant force is almost straight downwards. The end result of this arrangement is that the tank ends up being pulled up and the shuttle is pulled down. The very small values for $\mathcal{E}_s$ on **Figure 3**, and for **a** on **Figure 4**, for the 10–11 kilometer range of separation, and the subsequent large jump at the 12 kilometer point are related to the relative positions of the tank and shuttle at the time the tension is "turned on". As the external tank and shuttle drift apart after initial separation, the relative velocity between the two gradually starts to build, but it is not until the distance reaches the 12 kilometer point that the force on the tether is insufficient to eliminate the relative motion. After the twelve kilometer point, the energy would continue increasing proportional to the increases in separation distance except that the relative positions of the tank and shuttle

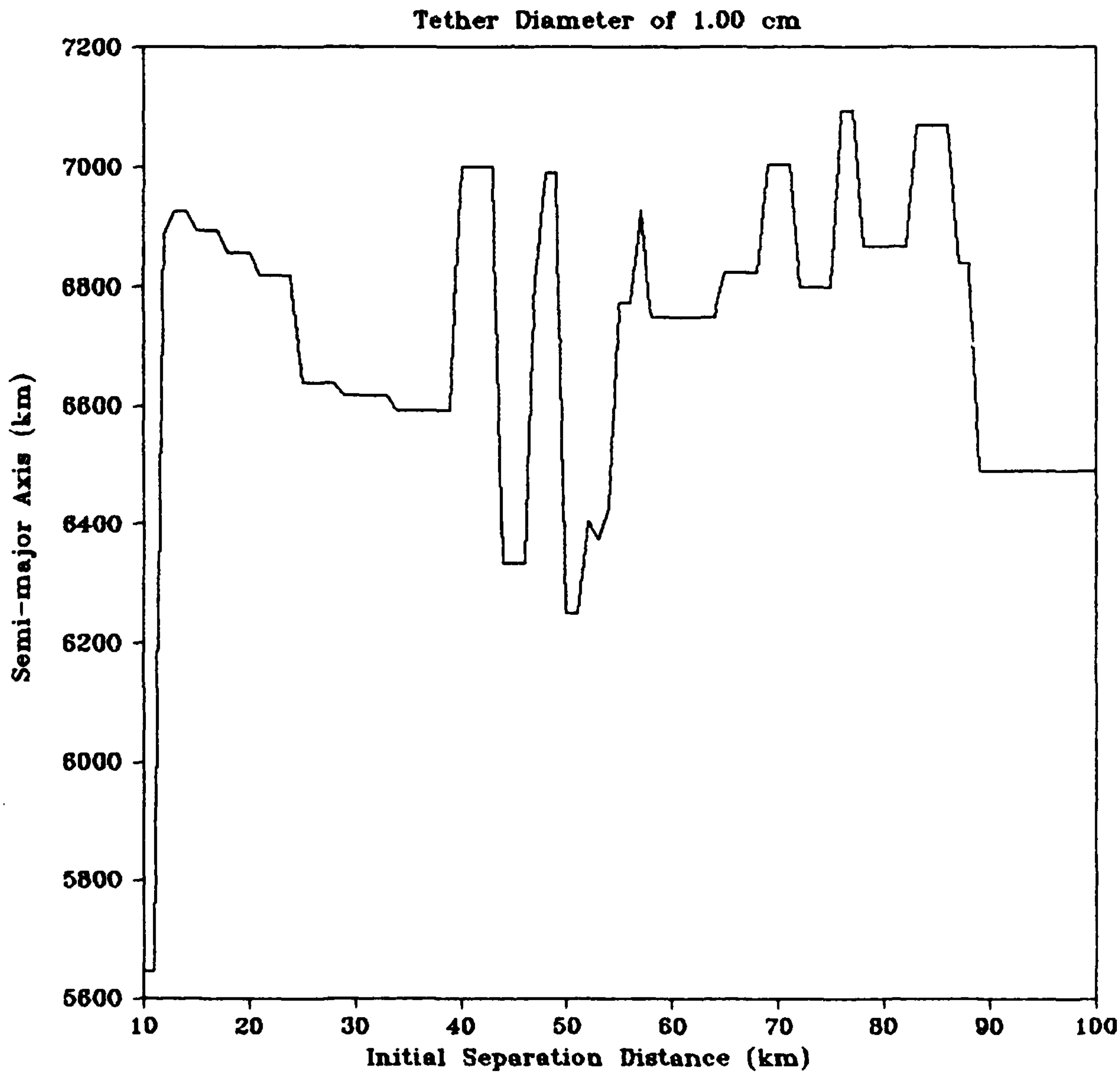

Figure 4 Initial Separation versus Semi–Major Axis

cause the external tank to impact on the earth before the tether reaches it's full deployment length. As the separation distance increases, more of the relative motion is in the horizontal mode and when the tether is tensioned the resultant force is more in the horizontal direction than in the vertical direction. However, since the external tank is at it's apogee when the initial separation occurs every bit of momentum that is transferred to the shuttle makes the descending path towards perigee that much steeper, and the transfer of momentum lowers the perigee of the tank from it' initial value at the separation point. Thus, as the initial separation distance increases in the 13 to 26 kilometer range the external tank has dropped that much lower in altitude before the shuttle can start using it as an energy source. It becomes a vicious circle because the longer the separation distance is the closer the tank is to the earth, and the less momentum transfer is required to cause it to crash. However, as the tether separation distance starts to increase further, another factor enters into the picture that tends to mitigate this problem. As the tether separation distance passes about the 28 kilometer point, the displacement from the local vertical that causes the restoring force in the tether becomes so large that the shuttle is actually, in some cases, slung ahead of the external tank. This is referred to as a swinging release (11:167), and can either increase or decrease the orbital energy of the shuttle. Of course, in most cases it is assumed that the tether will be released before it will start to decrease the energy of the payload. Thus the reason for the continued drop of both **Figures 3** and 4 after the 30 kilometer point is because the point of initial tension application is still far enough from the 100 kilometer termination point that the tether actually starts to pull the shuttle back down towards the external tank. The

orientation of the shuttle and the tank at the 40 kilometer separation point is such that the shuttle is released on the "upswing" of the momentum transfer from the external tank, and thus it receives the full benefit of the tank's energy. Prior to this point, the shuttle is released after passing the local vertical because the tether does not reach full deployment until that time. The large drop at the 45 kilometer point is because the tank crashes into the earth. This same factor is the reason for the large drop in the graphs at the 50 to 54 kilometer separation distances. By the time the 60 kilometer separation distance is reached, the tank is too far from the shuttle to drop the tank into the earth before the length limit is reached. The jumps that occur at the 65, 75, and 85 kilometer points are there because of the orientation of the tether at the moment of shuttle release. The shuttle is released prior to passing the local vertical and so does not lose the energy gained by the momentum transfer.

It is obvious that the maximum momentum transfer from the tank into the shuttle is a balancing act between pulling on the cable too soon and dropping the external tank into the earth (and the shuttle also) or slinging the shuttle around the tank and back towards the earth and a lower orbit or pulling too late and not getting the full benifit from the tether before maximum separation distance is reached. The different types of tether releases have been given names that are descriptive of the method in which the cable is allowed to pay out. When the tether is let out very slowly, and the two objects stay very close to the local vertical such that it appears that the bottom object is falling while the top one is climbing along the reference defined by the local vertical, this is called a "hanging" release. When the tether is allowed to pay out very fast and then tightened up so that the

shuttle is spun around at the end of the tether, this is called a "swinging" release. The increase in orbital energy between the "swinging" release (the jump points **Figure 3**), and the normal or "hanging" releases was by about a factor of 1.42 for the separation ranges between 50 and 86 kilometers. Previous attempts to quantify the contributions of tethers (13, 11, 12), have shown that the change in velocity (or "characteristic velocity") of a tether is a function of it's material properties, it's length, and the type of release used. For a swinging release with the shuttle released at or prior to the local vertical point of the tether, the effective increase in the velocity is by about a factor of 1.2 . Since the orbital energy is directly proportional to the square of the velocity, then any change in the velocity by a factor of 1.2 should give a proportional increase of the orbital energy by a factor of 1.44, with all other factors being the same. This agrees very well with the result that was given by the program for the jump from the value for $\mathcal{E}_s$ at an initial separation of 76 kilometers as opposed to 78 kilometers. Thus the jumps in the curves for **Figures 3** and 4 may be explained as being caused by the variations in either the initial or release geometry between the shuttle and external tank. The flat portion of the graphs in **Figures 3** and 4 starts at 90 kilometers, and continues until the distance reaches 100 kilometers. The reason for this flat section is that the tether did not have time to significantly change the orbit of the shuttle before the maximum tether length was reached. The values shown for this portion of the graph for $\mathcal{E}_s$ on **Figure 3**, and a on **Figure 4** is about the same as that for the unmodified orbit that was used as a test for the equations of motion of the system described in Section IV.

A comparison of rocket enginess presently being used to boost the shuttle with the *effectiveness of* the tether "toss" would show the benifits of tether useage. The way to accomplish this comparison would be to compare the effective specific impulse of the tether to that of the shuttle manuevering engines, and a typical solid fuel booster. To determine the specific impulse of the tether, it is first required to find the change in linear momentum of the shuttle at the time of tether release from it's pre— tether state. Then, the gain in shuttle performance measured as an effective I_{sp} will be determined by the ratio of increase in linear momentum to the penalty caused by having to carry the weight of the cable. This can be expressed by the following formula as:

$$I_{sp} = \Delta P / W_s \tag{37}$$

Where I_{sp} is the specific impulse of the tether, ΔP is the change in the linear momentum of the shuttle, and W_s is the weight of the cable. Since the material for the cable was chosen to be Kevlar the density, ρ_t , is 1.45 grams per cubic centimeter. Thus for a 100 kilometer tether with a diameter of 1.00 centimeter, the mass is 1.138827337×10^4 kilograms. Using the standard acceleration for gravity at the surface of the earth, the value of the I_{sp} for the entire range of separation distances shown in **Figures 3 & 4** was plotted in **Figure 5**. While the shape of the curve showing the performance of the tether does not change very much from the previous figures, there is additional information to show the relative I_{sp} of the shuttle

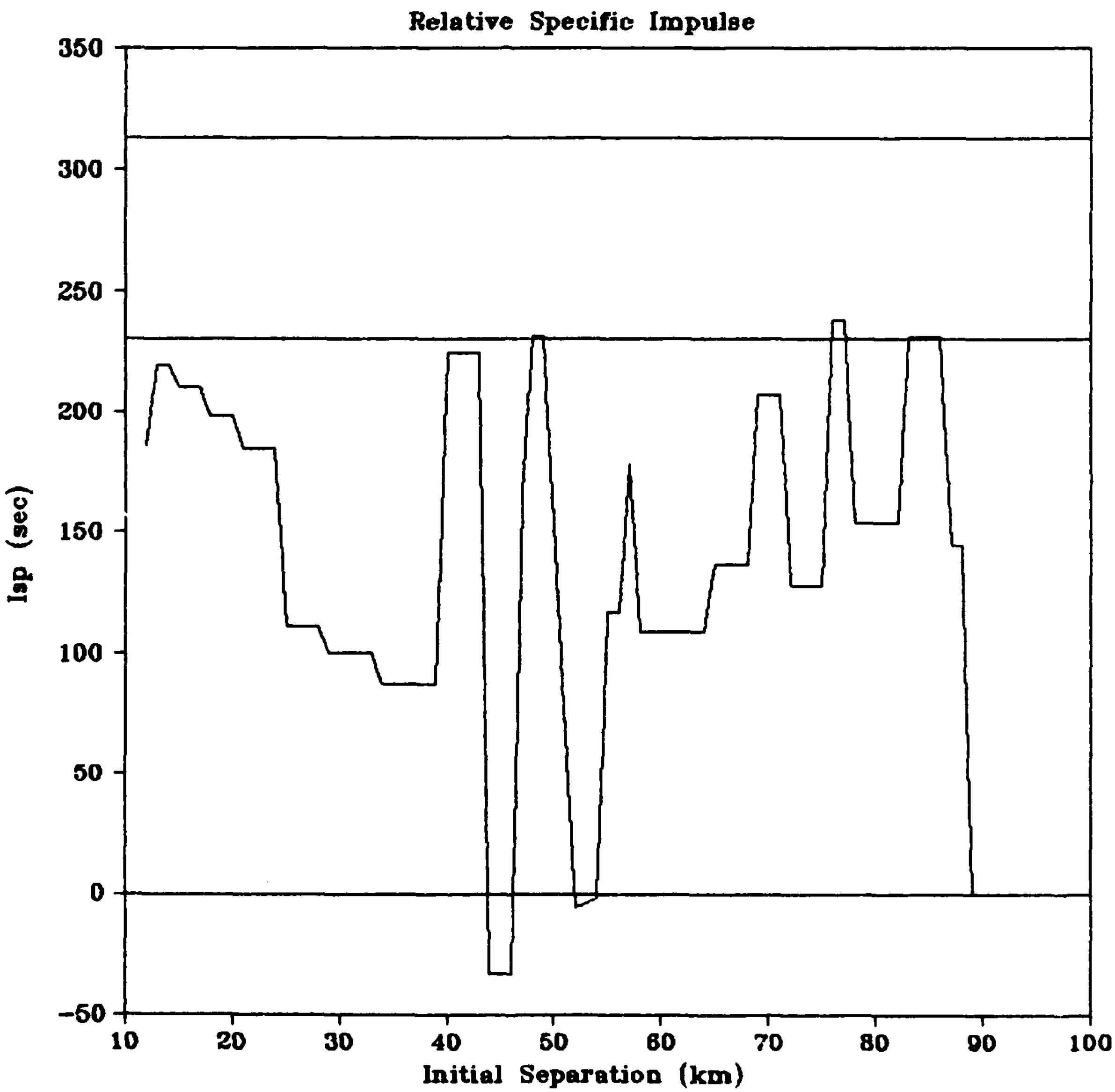

Figure 5 Specific Impulse Comparison

manuevering engines (I_{sp} of 313 seconds), the solid fuel booster (I_{sp} of 230 seconds), and the tether's I_{sp} (variable). The bottom line on the graph at zero represents the unaided (or normal) level with no added boost. The maximum specific impulse that the tether acquired was 237 seconds as conmpared to the 313 for the manuevering engines, and 230 for the solid fuel boosters.

The program based on the set of equations developed in Sections II and III was used to examine the range of release values between 10 and 100 kilometers, and determine the distance at which to apply tension that would result in the maximum altitude gain for the shuttle. The optimization routine determined the separation value that gave the maximum energy to the shuttle was for the distance of 76.742 kilometers. This agrees with the approximate point picked from the output (see Figures 3 and 4) of a different program that simply did an iterative search throughout the entire range of allowable tether values, and calculated the resulting orbital energies and semi–major axes. Using the value found by the optimization routine for the initial separation distance, the shuttle would acquire a semi–major axis, a , of 7093.129 kilometers and an orbital energy, $\mathcal{E}_s$, of -28.0977 km^2/sec_2 from a controlled tether "toss". This represents an increase in altitude of 603.066 kilometers from using a tether that is 100 kilometers long. The comparison of specific impulse with other types of shuttle propulsive devices enabled a comparison to be made of how well the tether performed relative to other currently avliable devices.

VI. Suggestions and Recommendations

The equations of motion have been developed and validated for calculating the motion of the combined external tank and space shuttle system while they are connected by a tether. Using the optimization procedure that was developed in Section III it is possible to determine the separation distance between the tank and the shuttle for application of tension that will result in the maximum altitude gain for a cable with given physical size and material properties. While there are many possible combinations of tether length and diameter, the results for a typical cable of Kevlar that is 100 kilometers long and 1 cm in diameter was used. The resulting "toss" contribution to the shuttle boost from the cable was maximized at an initial separation distance of 76.742 kilometers. The shuttle was boosted so that it's semi–major axis was increased from 6490.063 kilometers to 7093.129 kilometers, which is a gain of 603.066 kilometers.

The benefits from using the cable to transfer momentum from the external tank to the shuttle are obvious as this will increase the number of mission options that are available for the shuttle to fly. The increase in altitude that may be gained from a tether "toss" could be used to allow

rescue of malfunctioning satellites in medium altitude orbits that are presently impossible for the shuttle to reach. Not only rescue, but launch of satellites would benifit from the increase in shuttle altitude since medium altitude orbits could be reached without the requirement for a space tug to carry the satellite the last 400 km. In addition, by changing the optimization search routine, the program could be made to find the maximum increase in shuttle cargo weight that would still allow reaching the normal shuttle operating altitude. The extra velocity needed to carry the extra weight to orbit would come from the exchange of momentum with the external tank.

While the use of such a tether to improve the performance of the shuttle shows great promise, there are disadvantages as well as the advantages already listed. The disadvantages are increased complexity to control the tether tension, added weight for the tether and control mechanism, abort options in the event the tether breaks during deployment, and control of tank for disposal. The magnitude of the contribution that can be made to shuttle operation during the boost phase from a tether "toss" is enough to overcome all of the above disadvantages except the last. The momentum extraction from the external tank will cause it to fall in areas that have previously been considered safe from tank impact. Thus, tether disposal of the external tank appears to be a very promising method of operation for orbital boost, but control stability and guidance of the tank dispersion after use are issues that must be solved before the benefits may be realized.

While the development of the optimization algorithm is general enough to determine the performance of any size (length and diameter) tether, the

issue of how the tether compares on a cost basis compared to an equivalent rocket boost has not been addressed. It should be possible to determine approximate masses for the tether, deployment mechanism and control system to allow for a direct comparison on a dollar basis. In addition, now that the dynamics of the system have been determined it would be possible to specify a desired set of end conditions (in terms of orbital elements) for the shuttle, and using the optimization routine contained herein to determine the size of cable and tension profile required to meet those performance conditions.

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VITA

Captain Robert R. Fisher was born on 22 December 1953 in Dallas, Texas. He graduated from high school in Lake Jackson, Texas, in 1972 and attended the United States Air Force Academy from which he received the degree Bachelor of Science in Astronautical Engineering in 1976. Upon graduation, he received a regular commission and attended Undergraduate Pilot Training at Laughlin AFB, Texas. He received his wings in August 1977, and then served as a KC – 135 pilot and flight instructor at Minot AFB, N.D. and Beale AFB, CA. until entering the School of Engineering, Air Force Institute of Technology in May 1985.

Permanent Address: 528 Oak Drive
Lake Jackson, Texas 77566

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Abstract

OPTIMAL SHUTTLE ALTITUDE CHANGES USING TETHERS

The possible use of tethers in space has been proposed for the last hundred years. While much work has been done recently on the use of tethers for towed satellites from the Space Shuttle, little has been done to determine the possible benefits of using tethers as propulsive devices to supplement or replace rocket engines for boost from Low Earth Orbit. This project attempts to determine one method of using tethers to improve the performance of the Space Shuttle. Orbit insertion parameters such as velocity and final altitude for the space shuttle are limited by operational constraints on the possible delta V that can be supplied from the engines. The possibility of increasing the performance of the shuttle exits by use an inter–connecting tether to serve as a momentum transfer device between the External Tank and the Shuttle. This added momentum would widen the possible orbit options presently available by boosting the shuttle to a higher orbit. This project derives the equations of motion for a three–body connected dynamical system to include the Shuttle, the external tank, and the cable in orbit around a spherical Earth. Due to current material limitations the tether length is limited to 100 kilometers. The possible envelope of orbital changes is investigated, and this program determines through an optimization routine the tension profile in the cable, and the initial separation distance to apply tension to the cable that results in the maximum altitude gain for the shuttle.

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